MULTIMEDIA UNIVERSITY

STUDENT ID NO

SEAT NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (TERM ID 2220)

TMA1301 – Computational Methods

(All sections / Groups)

5 JULY 2023
2.30pm – 4.30pm
(2 Hours)

Question #	Marks
1	
2	
3	
4	
Total	

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **11** pages with **4** questions only excluding the cover page.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the **QUESTION BOOKLET**.

4. **You are required to write proper steps.**

QUESTION 1 [10 MARKS]

(a) Use **SIX DIGITS with rounding** for all workings.

Given the function $f(x)=\dfrac{\sin x}{x}-\cos x$.

(i) By using the first two nonzero terms in Taylor series expansion, convert the given $f(x)$ to a function that will avoid loss of significance, $g(x)$.

[HINT: $\sin x = x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-...;\quad \cos x = 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-...$] **[1.5 marks]**

(ii) Approximated $g(0.125)$. **[1 mark]**

(iii) The actual value of $f(0.125)=0.0052$. Find the relative error from the result obtained in (ii). **[0.5 mark]**

Continued...

(b) Use **FOUR decimal places** for all workings.
Consider the function $f(x) = x^3 - 2x^2 + x$

(i) Find $f'(x)$. Then perform **THREE** iterations using **Newton's method** to reach an approximated root, starting with initial value $p_0 = 0.9$. Complete the following table.

[Hint : $P_n = P_{n-1} - \frac{f(P_n)}{f'(P_n)}$] **[3.5 marks]**

(i)

n	p_{n-1}	$f(p_{n-1})$	$f'(p_{n-1})$	p_n
1				
2				
3				

Continued...

(ii) If the root of $f(x)$ is $r = 1$ and Newton's method is employed to find the root, does the convergence exhibit linear or quadratic? **[1 mark]**

(iii) Based on the answer obtained in (ii), find the convergence rate. Hence express the error e_{n+1} in terms of the previous error e_n, where e_n is the error for *nth* term. **[2.5 marks]**

Continued...

QUESTION 2 [10 MARKS]

Use **FOUR decimal places** for all workings.

Given the definite integral $\int_2^3 \frac{1}{x+1}\,dx$.

(a) Find the actual value for the definite integral. **[1 mark]**

(b) Find the approximated values for the definite integral using the (i) Trapezoidal Rule and (ii) Simpson's Rule. For each, divide the range into four equal parts. **[4 marks]**

(c) Find the absolute errors by using the results obtained from part (a) and (b). **[1 mark]**

(d) Find the number of subintervals if Composite Trapezoidal Rule is used to approximate the definite integral with an error of at most 10^{-4}.

[Hint : $-\frac{b-a}{12}h^2 f''(\xi)$ for $a \le \xi \le b$] **[4 marks]**

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QUESTION 3 [10 MARKS]

(a) Perform LU Factorization for the following matrix, W.

$$W = \begin{bmatrix} 2 & -10 & 12 \\ 1 & -4 & 6 \\ 3 & -15 & 17 \end{bmatrix}$$

[3 marks]

Continued...

(b) Find the eigenvalues for the following matrix, M.

$$M = \begin{bmatrix} 10 & 12 \\ 15 & 18 \end{bmatrix}$$

[2.5 marks]

(c) Solve for x, y, z using the Gauss-Siedel Method in **two** iterations for the following systems of linear equations. Use the initial values of $x = 1, y = 0, z = 2$ and **four decimal places** for all workings.

$$\begin{aligned} 13x + y - 4z &= 5 \\ -15x + 20y + 4z &= 3 \\ x - 10y + 12z &= 4 \end{aligned}$$

[4.5 marks]

Continued...

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QUESTION 4 [10 MARKS]

[**Note:** For part (a), use **three decimal places** for all workings.]

(a) The following table shows the population of Malaysia from year 2017 to 2022.

Year	2017	2018	2019	2020	2021	2022
Population (in million)	31.975	32.399	32.804	33.200	33.573	33.975

The population of Malaysia is assumed to have an exponential growth. The equation of an exponential function is given as $y = ae^{bx}$.

(i) The exponential function can be linearized in the form of $Y = A + bx$ by taking logarithm. Find the expressions for Y and A. **[1.5 marks]**

(ii) Complete the following table. **[2 marks]**

Year	x	y	x^2	Y	xY
2017	1	31.975			
2018	2	32.399			
2019	3	32.804			
2020	4	33.200			
2021	5	33.573			
2022	6	33.975			
	$\sum x = 21$		$\sum x^2 = 91$	$\sum Y =$	$\sum xY =$

Continued...

(iii) From (i) and (ii), find the exponential function of the form $y = ae^{bx}$ using the **method of least squares.**

[Hint: $a = \dfrac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$, $b = \dfrac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$]

[2 marks]

(iv) From (iii), estimate the population (in million) of Malaysia in **year 2026.**

[0.5 mark]

Continued...

[**Note:** For part (b), use **four decimal places** for all the workings.]

(b) Given a function $f(x) = e^{-2x}$.

(i) Find the **second degree Mclaurin Polynomial** from the given $f(x)$. **[3 marks]**

(ii) From (i), approximate $f(0.25)$. **[0.5 mark]**

(iii) Find the **absolute error** for the approximation obtained in (ii). **[0.5 mark]**

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